Evaluation Method of Polynomial Models' Prediction Performance for Random Clock Error

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In satellite navigation systems such as the Global Positioning System, clock error is one of the major sources of error in precise pointing. In order to remove clock error, it is modeled as a second-order polynomial and the clock-error correction parameters are sent to users. However, a random clock error cannot be modeled as a second-order polynomial. Therefore, the time discrepancies due to random clock error must be taken into consideration for precise pointing. This paper proposes an analytical computation method for estimating the random clock error in the current system which makes use of the Allan variance characteristics of random clock error without random clock realization and a lot of simulation studies. Moreover, a numerical example based on the proposed method shows that the first-order polynomial model is better for predicting a random clock error than the second-order polynomial.

I. Introduction

IN recent years, the Global Positioning System (GPS) has been developed as a means of satisfying demands in various fields for near-real-time, high-accuracy navigation.[1-3] In satellite navigation systems such as the GPS, there are numerous error factors that influence precise point positioning and one of the major ones is clock error. This is so because, in user positioning calculation, each GPS satellite position is determined using the signal transmission time $t_s = T_s + \mathrm{d}t_s$, where T_s is the satellite clock time and $\mathrm{d}t_s$ the satellite clock offset due to an atomic clock error. Therefore, the users' positioning accuracy is extremely dependent on the accuracy of the clock-error estimated $\mathrm{d}t_s$. In order to remove these effects, the current GPS system corrects the clock error by using the second-order polynomial model, that is,

$$\mathrm{d}t_s = d_0 + d_1(t - t_r) + d_2(t - t_r)^2 \tag{1}$$

where t_r is reference time. The clock-correction parameters d_0, d_1, d_2, and t_r are sent to users as navigation messages from each GPS satellite. However, even if these clock-error correction parameters are used, the true time at data transmission cannot be obtained. The objective of this paper is to evaluate the accuracy of the clock-error correction with the second-order polynomial model currently used in the GPS system.

Most users seem to know the accuracy of the clock-error correction in assessing the positioning accuracy in their own GPS applications. Generally, the evaluation of estimation error using polynomial models should be done with actual data (satellite clock data). Although GPS users do not have access to actual GPS clock data, but they can use the Allan variance characteristics of a GPS satellite clock, which is a measure of the second-moment properties of clock performance. There are two approaches to the evaluation of clock-error correction based on the Allan variance characteristics. One is to realize random clock data from an Allan variance and to evaluate the polynomial fitting error. The other is to estimate directly the correction error variance using an Allan variance. For the former method, random clock realization algorithms have been proposed by several people including Poole.[5] With random clock data realization, sample cases can be evaluated. However, to evaluate the total characteristics of clock-error correction, a lot of sample cases must be examined. The latter method can result in the total evaluation of clock-error correction by simple analytical calculations without random clock realization and numerous simulation studies. The method proposed in this paper fits into the category of the latter evaluation method. It employs the results of Fell's random clock-error analysis[4] for calculating covariances of random clock error. A numerical example using the proposed method shows the characteristics of the random clock-error correction of atomic clocks using the polynomial model.

GPS Navigation Problem and Clock Error

Let us consider the case of range measurements and formulate the GPS navigation problem. The measurement of GPS users from GPS satellite i is a pseudorange, which can be expressed by

$$R_i = r_i + c(\mathrm{d}t_u - \mathrm{d}t_{si}) + c(\mathrm{d}t_{Ai}) + n_i \tag{2}$$

$$r_i = \sqrt{(x_{si} - x)^2 + (y_{si} - y)^2 + (z_{si} - z)^2} \tag{3}$$

$$R_i = c(T_R - T_{si}) \tag{4}$$

where R_i is the observed pseudorange, r_i the true range, c the light speed, $\mathrm{d}t_u$ the user's clock offset from GPS time, $\mathrm{d}t_{si}$ the GPS satellite i clock offset, $\mathrm{d}t_{Ai}$ the signal propagation delay due to atmospheric effect, n_i the random range measurement error due to receiver noise, etc., (x_{si}, y_{si}, z_{si}) the satellite position at t_{si}, (x,y,z) the user position at t_R, t_{si} the signal transmission time in GPS system time, t_R the signal receiving time in GPS system time, t_R the signal receiving time in GPS system time, T_{si} the signal transmission time in satellite clock time, and T_R the signal receiving time in receiver clock time.

The user must solve for four unknowns in these observation equations: his position coordinates x,y,z and his clock offset $\mathrm{d}t_u$. Therefore, a user need not use a precise instrument such as an atomic clock because his clock error can be estimated. In order to obtain these four unknown values, more than four observation equations are needed. There are several unknown parameters in observation equations. The user estimates $\mathrm{d}t_{Ai}$ by measuring the pseudo-ranges at two points: the L_1,L_2 fre-

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quency and the ionosphere delay correction. The x_{si}, y_{si}, z_{si}, and dt_{si} must be computed by the user from the navigational information provided by the GPS satellites. In the user's positioning calculation, x_{si}, y_{si}, z_{si}, and dt_{si} are assumed to be true values. However, x_{si}, y_{si}, and z_{si} are calculated by using the signal transmission time $t_{si}=T_{si}+dt_{si}$. Therefore, a user's navigation accuracy is very dependent on the accuracy of his clock-error estimate dt_{si}. In other words, if not corrected, the clock error significantly affects the navigation accuracy.

Consider a typical atomic clock whose frequency is represented by the following model:

$$f_i(t)=f_I+df+\dot{f}\cdot(t-t_0)+\tilde{f}(t) \tag{5}$$

where f_I is the frequency of an ideal clock, df a frequency bias, $\dot{f}$ a drift in frequency, $\tilde{f}(t)$ a random fluctuation in frequency, and t_0 a clock reset time. A clock records N_i cycles in time interval (t,t_0) where

$$N_i=f_I\cdot(t-t_0)+df\cdot(t-t_0)+\frac{\dot{f}\cdot(t-t_0)^2}{2}+\int_{t_0}^{t}\tilde{f}(s)ds \tag{6}$$

In addition, the ideal and real clocks may not be synchronized at t_0, introducing a time or phase error at t_0 as $dN_0\cdot T_I$. Then, t_i is given by

$$t_i-t_0=(t-t_0)+\frac{df}{f_I}(t-t_0)+\frac{\dot{f}}{2f_I}(t-t_0)^2$$

$$+\frac{1}{f_I}\int_{t_0}^{t}\tilde{f}(s)ds+dN_0\cdot T_I \tag{7}$$

where t_i is the time of the atomic clock and T_I that of the ideal clock. From this relationship, the time error at t is represented by

$$dt_s(t)=A_0+A_I\cdot(t-t_0)+A_2\cdot(t-t_0)^2+x(t) \tag{8}$$

$$x(t)=\int_{t_0}^{t}y(s)ds \tag{9}$$

where $y(s)=\tilde{f}(s)/f_I$ and $x(t)$ is a random clock error, A_0 a time bias, A_1 a time drift, and A_2 an aging term.

Therefore, the current GPS system corrects the clock error by using the second-order polynomial model. However, the clock error also contains a random error that cannot be sufficiently accounted for with the second-order polynomial model. Predictions of random clock error made by the second-order polynomial model should be evaluated for precise pointing. Although most users cannot directly evaluate the satellite clock error with the actual satellite clock data, they can use the Allan variance characteristics to assess the satellite clock error. The purpose of this paper is to derive a method for evaluating the prediction of the satellite clock error using only an Allan variance, without resorting to random clock realization and/or numerous simulation studies.

Estimation Error Variance of Polynomial Model Fitting

Assume that $x(t)$ is a random clock error and the estimate $x(t)$ is given by the mth-order polynomial model

$$x(t)=\sum_{i=0}^{m}a_i(t-t_r)^i \tag{10}$$

As shown in the next section, according to the characteristics of the random clock error, $x(t)$ and $x(t+s)$ are closely correlated if s is small. Therefore, polynomial model fitting is effective to predict random clock error. The estimate of coefficients a_i can be obtained by minimizing the criterion,

$$L=\int_0^T\left[x(t)-\sum_{i=0}^{m}a_i(t-t_r)^i\right]^2dt\to\min$$

or

$$L=\sum_{j=1}^{n}\left[x(t_j)-\sum_{i=0}^{m}a_i(t_j-t_r)^i\right]^2\to\min$$

where $[x(t_1),\ldots,x(t_n)]$ is sampling data in a time interval T and t_r reference time. For the sake of computational simplicity, the discrete-type estimation problem is considered here. Thus, the estimates of the coefficients in the polynomial model can be obtained by using the least-square method,

$$(a_0,\ldots,a_m)^T=(A^TA)^{-1}A^T\cdot[x(t_1),\ldots,x(t_n)]^T \tag{11}$$

where

$$A=\begin{bmatrix}1, & (t_1-t_r), & \ldots, & (t_1-t_r)^m\\ \\ 1, & (t_n-t_r), & \ldots, & (t_n-t_r)^m\end{bmatrix}$$

By setting the matrix $C=(A^TA)^{-1}A^T$ where $C=(c_{ij})$, the coefficients can be expressed by

$$a_i=\sum_{j=1}^{n}c_{i+ij}x(t_j) \tag{12}$$

Therefore, the estimate of random clock error at $t,\hat{x}(t)$ is

$$\hat{x}(t)=\sum_{i=1}^{n}e_ix(t_i) \tag{13}$$

and

$$e_i=\sum_{j=1}^{m+1}c_{ji}(t-t_r)^{(j-1)} \tag{14}$$

From this, the estimation error is

$$\tilde{x}(t)=x(t)-\hat{x}(t)=Z\cdot X^T$$

where

$$Z=(1,-e_1,-e_2,\ldots,-e_n)$$

$$X[x(t),x(t_1),\ldots,x(t_n)]$$

The statistics of the estimation error can be calculated as follows. First, the mean of the estimation error becomes

$$E[x(t)]=E(Z\cdot X^T)=Z\cdot E(X^T) \tag{15}$$

and the variance of estimation error $V(t)$ is

$$V(t)=Z\cdot R(t,t_1,t_2,\ldots,t_n)\cdot Z^T \tag{16}$$

where R is a $(n+1)\times(n+1)$ matrix whose (i,j) element is $r(t_{i-1,j-1})$. $r(t_i,t_j)$ is the correlation of $x(t_i)$ and $x(t_j)$ [that is, $r(t_i,t_j)=E(x(t_i)x(t_j))$] and t_0 is defined to be t. That is,

$$R(t,t_1,t_2,\ldots,t_n)=\begin{bmatrix}r(t,t), & \ldots & r(t,t_n)\\ & r(t_i,t_j) & \\ r(t,t_n) & \ldots & r(t_n,t_n)\end{bmatrix} \tag{17}$$

If the correlation of random clock error $r(t_i,t_j)$ can be obtained analytically, then the estimation error of the polynomial-fitting variance $V(t)$ can be calculated without random clock data realization or simulation studies. Fell's results give the approximate correlation $r(t_i,t_j)$ from an Allan variance. His method is utilized for computing Eq. (16) and is summarized in the next section.

Correlation of Random Clock Error Using an Allan Variance

According to the previous investigations, a random clock error $x(t)$ is given by the integral of the instantaneous fractional frequency deviation $y(t)$ and an Allan variance $\sigma_y^2(t)$ is defined by

$$\sigma_y^2(t) = \tfrac{1}{2}E[(Y_{m+1} - Y_m)^2] \tag{18}$$

$$Y_m = \frac{1}{N}\sum_{k=m}^{(m+1)N-1} y_k \tag{19}$$

or

$$Y_m = \frac{1}{t}\int_{t_m}^{t_{m+1}} y(s)\mathrm{d}s \tag{20}$$

where $t_{m+1} = t_m + t$ and the expectation $E(\cdot)$ should be an infinite time average. An Allan variance for a satellite oscillator is shown in Fig. 1, which plots white noise, flicker noise, and the integral of white noise. Using the Allan variance, the correlation of random clock error $x(t)$ can be calculated approximately by performing the following integral calculation:

$$E[x(t_i)x(t_k)] = \sum_{j=1}^{5}\int_0^{t_j}\int_0^{t_k}\sigma_j^2 e^{-\beta_j|s'-s|}\mathrm{d}s\mathrm{d}s' \tag{21}$$

Performing the above integral calculation, $r(t_i,t_k)$ can be derived as

$$r(t_i,t_k) = \sum_{j=1}^{5}\frac{\sigma_j^2}{\beta_j}\Big\{2(t_i - t_s) + \frac{1}{\beta_j}[e^{-\beta_j(t_j - t_s)} + e^{-\beta_j(t_k - t_s)} - e^{-\beta_j(t_k - t_i)} - 1]\Big\} \tag{22}$$

for t_k greater than t_i and where t_s is the start or reset time of the clock. The algorithm for obtaining σ_j and β_j from an Allan variance, which is described in Fell,[4] is summarized in the following procedure.

Step 1. From the Allan variance, the power spectral density $S_{yy}(\omega)$ shown in Fig. 2 can be obtained using the relationships of Table 1.

Step 2. $S_{yy}(\omega)$ is approximated by $S'_{yy}(\omega)$ which estimates the flicker noise by the second-order spectral density shown in Fig. 2. The definition of the three-stage transfer function approximating flicker noise is given in Table 2. The following results of the approximated power spectral density $S'_{yy}(\omega)$ are used:

a) Input data. t_1, t_2, t_3, and σ_f of Allan variance parameters are given as input data.

b) Intervals for piecewise approximation. The connecting points of the piecewise approximation function are

$$\omega_1 = 0, \omega_2 = W_1, \omega_3 = {}_aW_a, \omega_4 = a^3 W_a,$$

$$\omega_5 = a^5 W_a, \omega_6 = 1.0E4$$

where

$$W_1 = 6\ln 2/(\pi t_2), \quad W_2 = \pi/(2t_1\ln 2)$$

$$a = (W_2/W_1)^{1/6}, \quad W_a = W_1\sqrt{a}$$

$$N_0 = t_1\sigma_f^2, \quad N_1 = \pi\sigma_f^2/(2\ln 2), \quad N_2 = 3\sigma_f^2/t_2, \quad N_3 = \sigma_f^2 t_3^2/t_2$$

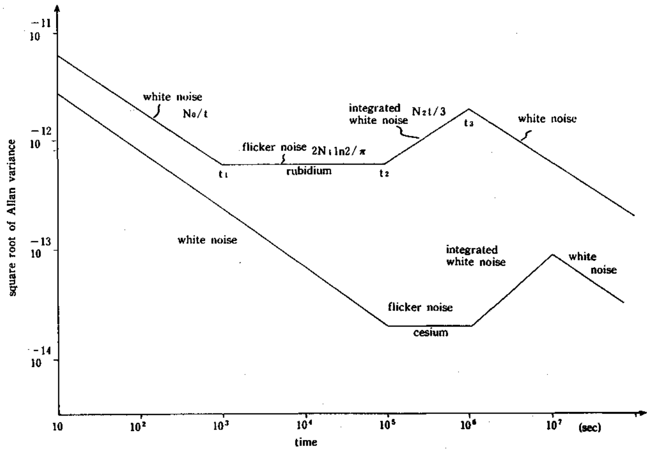

Fig. 1 Satellite and station oscillator's Allan variance (from Ref. 4).

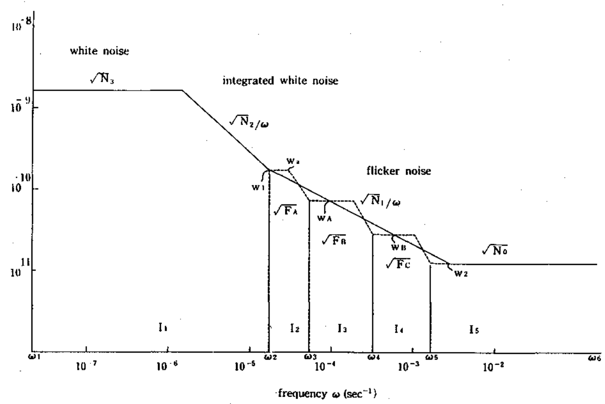

Fig. 2 Oscillator transfer function (square root).

Table 1 Allan variance and power spectral density

Error source $y(t)$	Allan variance $\sigma_y^2(t)$	Spectral density $S_y(w)$
White noise	N_0/t	N_0
Flicker noise	$2N_1\ln 2/\pi$	$N_1/\lvert w\rvert$
Integral of white noise	$N_2t/3$	N_2/w^2

c) The function values of the approximated power spectral density are

$$S'_{yy}(\omega_1) = N_3 \qquad S'_{yy}(\omega_4) = \frac{a^3\cdot W_1\cdot N_1}{a^6 W_a^2}$$

$$S'_{yy}(\omega_2) = \frac{N_1}{W_1} \qquad S'_{yy}(\omega_5) = N_0$$

$$S'_{yy}(\omega_3) = \frac{a\cdot W_1\cdot N_1}{a^2 W_a^2} \qquad S'_{yy}(\omega_6) = \frac{N_0}{100}$$

Step 3. The power spectral density $S'_{yy}(\omega)$ is approximated by the five piecewise first-order Marcov model S_j; that is,

$$S_j(\omega) = \frac{2\beta_j\sigma_j^2}{\omega^2 + \beta_j^2} \tag{23}$$

$S_j(\omega)$ is an approximation on the interval $I_j = (\omega_k, \omega_\ell)$ based on the following constraints:

a) At zero frequency, the approximating Markov power spectral density equals the second model at frequency ω_k

$$S_j(0) = S'_{yy}(\omega_k) \tag{24}$$

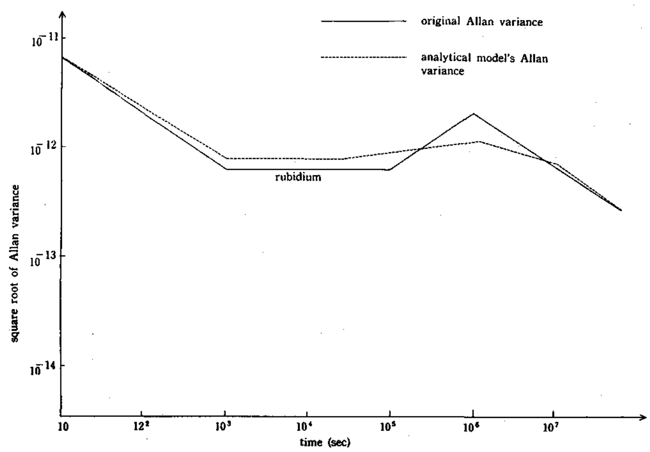

Fig. 3 Comparison of the approximated Allan variance and original Allan variance.

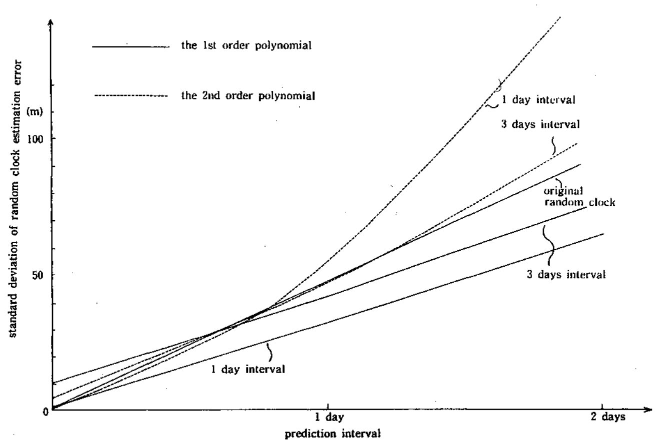

Fig. 4 Correction-error standard deviation of random clock.

b) In the limit as ω increases, the value of the function S_j converges to the following function:

$$\lim_{\omega \to \infty} S_j(\omega) = \frac{2\sigma_j^2 \beta_j}{\omega^2} \tag{25}$$

At ω_ℓ, this limiting value is set equal to the value of $S'_{yy}(\omega)$,

$$\frac{2\sigma_j^2 \beta_j}{\omega\ell^2} = S'_{yy}(\omega_\ell) \tag{26}$$

Equations (24) and (26) give the solutions of two unknowns, which are

$$\sigma_j^2 = [\omega_{j+1} \cdot \sqrt{S(\omega_j) + S(\omega_{j+1})}]/2 \tag{27}$$

$$\beta_j = \omega_{j+1} \cdot \sqrt{S(\omega_{j+1})/S(\omega_j)} \tag{28}$$

Step 4. The autocorrelation function $\phi_j(t)$ of $S_j(\omega)$ is

$$\phi_j(t) = \sigma_j^2 e^{-\beta_j |t|} \tag{29}$$

If we use σ_j and β_j, the covariance of instantaneous clock error $y(t)$ can be given by

$$E[y(t)y(t')] = \sum_{j=1}^{5} \sigma_j^2 e^{-\beta_j |t'-t|} \tag{30}$$

Equation (21) can be obtained from Eq. (30).

Table 2 Definition of three-state transfer function approximation

Function	Interval	Definition
F_A	$W_1 \le \omega \le W_a$	N_1/W_1
	$W_a \le \omega \le aW_a$	N_A/ω^2
	$aW_a \le \omega \le W_A$	$N_A/a^2W_a^2$
F_B	$W_A \le \omega \le a^2 W_a$	$N_A/a^2W_a^2$
	$a^2 W_a \le \omega \le a^3 W_a$	N_B/ω^2
	$a^3 W_a \le \omega \le W_B$	$N_B/a^6W_a^2$
F_C	$W_B \le \omega \le a^4 W_a$	$N_B/a^6W_a^2$
	$a^4 W_a \le \omega \le a^5 W_a$	N_c/ω^2
	$a^5 W_a \le \omega \le W_2$	N_0

where
$N_A = aW_1N_1, \quad N_B = a^3W_1N_1, \quad N_c = a^5W_1N_1$

$a = (W_2/W_1)^{1/6}, \quad W_a = W_1\sqrt{a}$

Table 3 Allan variance parameters and approximated piecewise correlation function

a) Allan variance parameters

t_1	1.0×10^3
t_2	1.0×10^5
t_3	1.0×10^6
σ_f	6.0×10^{-13}

b) Piecewise correlation function

Interval	σ_j^2	β_j
I_1 (0.0, 1.32E−5)	3.1E−24	1.73E−6
I_2 (1.32E−5, 4.79E−5)	6.26E−25	2.03E−5
I_3(4.79E−5, 2.67E−4)	6.26E−25	1.13E−4
I_4(2.67E−4, 1.49E−3)	6.26E−25	6.26E−4
I_5(1.49E−3, 1.0E4)	1.80E−19	1.0E3

V. Numerical Example

Allan Variance and an Approximated Correlation Function of Random Clock Error

In this numerical example, an Allan variance of the rubidium clock described by Fell[4] is employed. Based on the Allan variance parameters listed in Table 3a, the approximated power spectral density $S'_{yy}(\omega)$ and σ_j, β_j are calculated using the algorithm described in Eq. (4) and the results given in Table 3b. These values are used in the following numerical experiment.

Evaluation of Fell's Approximation

The calculated prediction error variance of Eq. (16) is very dependent on Fell's approximation in Eq. (4). In order to evaluate the performance of this approximation, the Allan variance corresponding to the approximate correlation function of Eq. (30) is computed. According to Eq. (18), the Allan variance $\sigma_y^2(t)$ is given as

$$\sigma_y^2(t) = \frac{1}{2} \cdot E\left\{\frac{1}{t^2}\left[\int_s^{s+t} y(s)\mathrm{d}s - \int_{s+t}^{s+2t} y(s)\mathrm{d}s\right]^2\right\} \tag{31}$$

With Eq. (31), the Allan variance corresponding to the approximated autocorrelation function becomes

$$\sigma_y^2(t) = \frac{1}{t^2}\sum_{j=1}^{5}\frac{\sigma_j^2}{\beta_j^2}(-e^{-2\beta_j t} + 4e^{-\beta_j t} + 2\beta_j t - 3) \tag{32}$$

The $\sigma_y^2(t)$ obtained with Fell's approximation and the original Allan variance are compared in Fig. 3. Based on these results, Fell's approximation seems to be a fairly good method for evaluating the prediction of random clock error.

Table 4 Standard deviation of range error due to random clock estimation error [$dR = cs(t)$]

Fitting interval		Estimation time point				
		t_r	$t_r + 30$ min	$t_r + 60$ min	$t_r + 90$ min	$t_r + 1$ day
First-order polynomial	0.5 h	0.1004	0.6074	1.1773	1.7826	36.9585
	1.0 h	0.1821	0.6751	1.2102	1.7725	34.9132
	1.5 h	0.2961	0.7595	1.2808	1.8239	33.8261
	3.0 h	0.5373	1.0241	1.5316	2.0531	32.1794
	1.0 day	4.4786	5.0838	5.6004	6.1246	33.0621
Second-order polynomial	0.5 h	0.0658	1.2593	3.6414	7.2178	1402.014
	1.0 h	0.1081	0.9384	2.3238	4.2646	665.442
	1.5 h	0.1540	0.8919	1.9987	3.4734	451.260
	3.0 h	0.3014	0.9566	1.7968	2.8192	241.153
	1.0 day	2.4855	3.0696	3.6921	4.3435	59.046
Random clock error (no estimation)		0.0658	1.2301	2.3571	3.4687	48.4046

Table 5 Fitting error of standard deviation of random clock estimation error

Fitting interval	First-order polynomial	Second-order polynomial
0.5 h	0.0245	0.7098
1.0 h	0.0179	0.3304
1.5 h	0.0139	0.2189
3.0 h	0.0092	0.1088
6.0 h	0.0067	0.0553
1.0 day	0.0049	0.0182

Examples of Standard Deviation of Range Error due to Prediction Error

Using Eq. (16), the standard deviation of range error due to clock-correction error is calculated. The first- and second-order polynomial models are employed in this experiment. The numerical examples are shown in Table 4. Numerical values of the range error are obtained by the equation of $dR = c \cdot s(t)$, where $s(t)$ is the standard derivation of the prediction error. The fitting intervals for estimating the polynomial coefficients are taken for several hours before a reference time t_r and at several time points after a particular reference time are evaluated to see the prediction performance of the polynomial models. From these results, the second-order polynomial model proves to be better than the first-order polynomial in the neighborhood of the reference time, but the second-order model is worse than the original random clock error if the predictive time point is far from the reference point. This means that the second-order polynomial model may cause significant error if the positioning time using GPS signals is far from the reference point. Comparing the results of the first- and second-order polynomial models, we see that the first-order polynomial is more accurate in predicting random clock error.

Linear Behavior of Standard Deviation of Prediction Error

The linear behavior of the standard deviation of random clock estimation error is also covered in this study. The standard deviation $s(t)$ for a long prediction interval has been evaluated and the result is shown in Fig. 4. The standard deviations of prediction error are almost linear if the first-order polynomial model is used. Therefore, it seems that the standard deviation of the prediction error can be approximated by the first-order polynomial model, that is,

$$\hat{s}(t) = c_0 + c_1 (t - t_r) \tag{33}$$

Several standard deviations of fitting error $\tilde{s}(t) = s(t) - \hat{s}(t)$ are evaluated in Table 5. These results show that the standard deviation of random clock prediction error is almost linear if a linear estimator is used, which can be observed directly from an Allan variance.

Conclusion

A method of evaluating the variance of the prediction error of random clock error using polynomial models has been derived using Fell's results on random clock analysis. This method can evaluate the prediction properties of random clock error using a polynomial model without random clock realization and numerous polynomial-fitting simulations. That is, if the Allan variance characteristics of an atomic clock are available, users can evaluate the expected variance of prediction error using polynomial fitting with very simple calculations. From the numerical experiments using the proposed method, it has been shown that the first-order polynomial model is better for predicting random clock errors that the second-order polynomial model and that the standard deviation of the prediction error is almost linear.

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